第 11.1 节 反常积分的概念

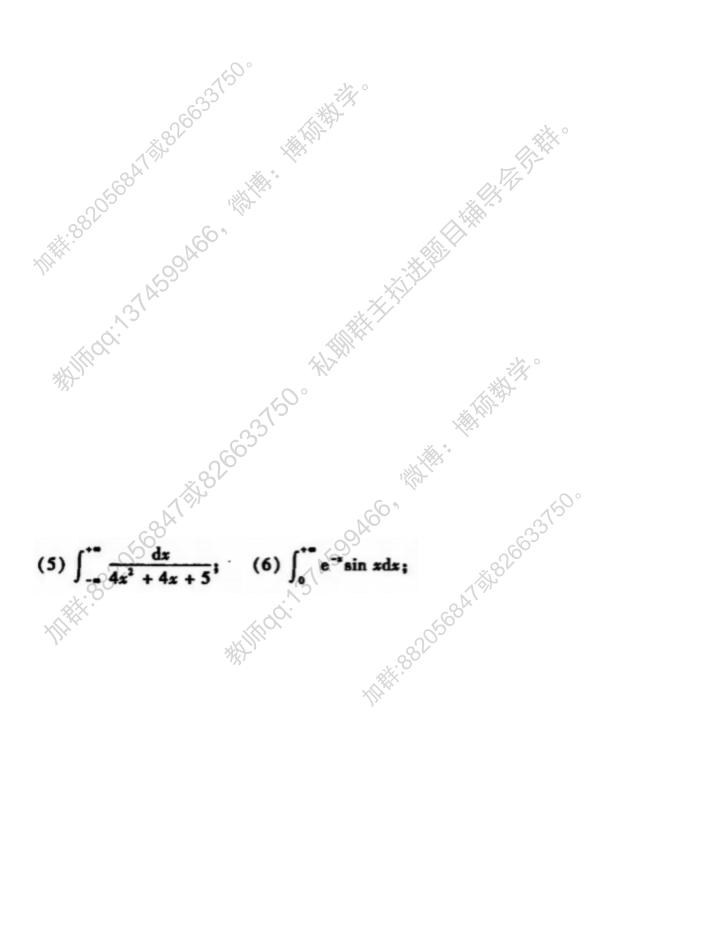
1. 讨论下列无穷积分是否收敛? 若收敛,则求其值:

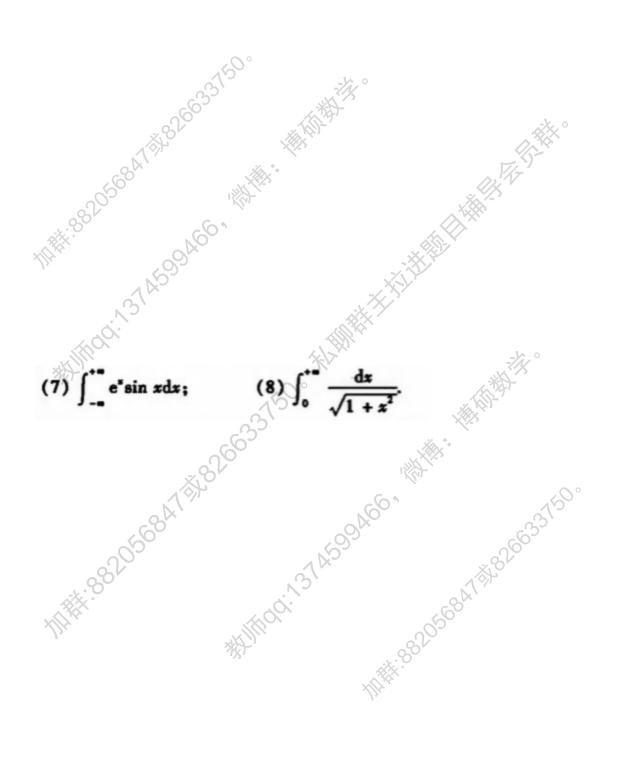
(1)
$$\int_0^{\infty} xe^{-x^2} dx$$
; (2) $\int_0^{\infty} xe^{-x^2} dx$;

White divide in the state of th (3) $\int_{0}^{\infty} \frac{1}{\sqrt{e^{\epsilon}}} dx;$ (4) $\int_{1}^{\infty} \frac{dx}{x^{2}(1+x)};$

$$(3) \int_0^{\infty} \frac{1}{\sqrt{e^x}} dx;$$

$$(4) \int_{1}^{+\infty} \frac{dx}{x^{2}(1+x)};$$





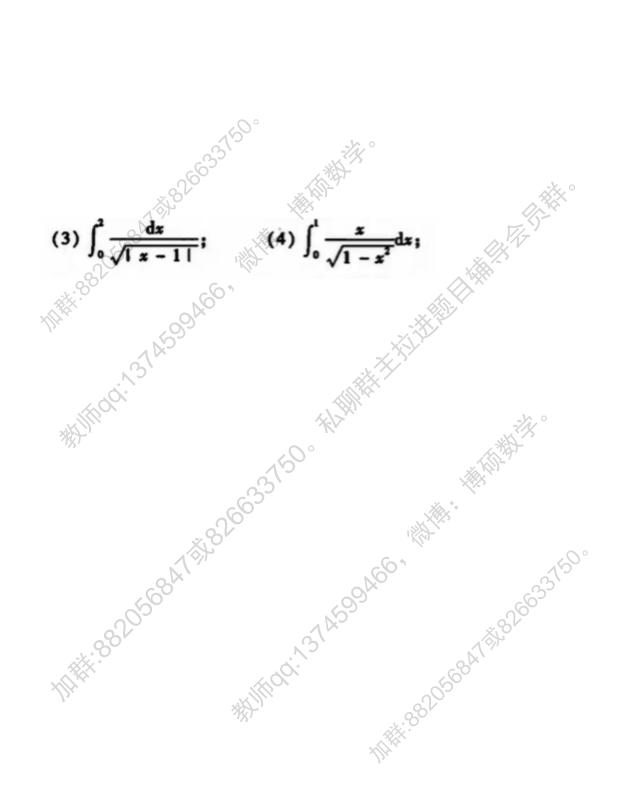
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2. 讨论下列瑕积分是否收敛? 若收敛,则求其值:

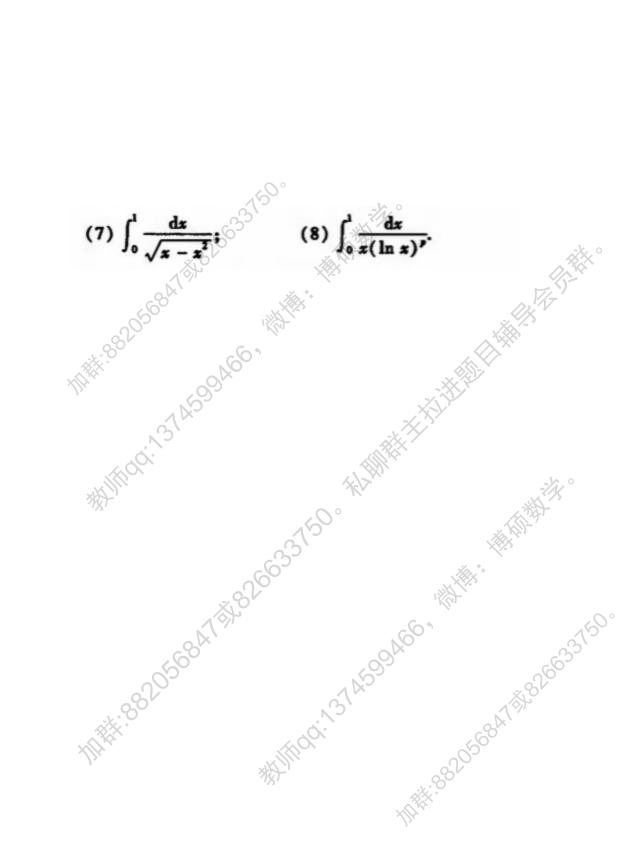
$$(1) \int_a^b \frac{\mathrm{d}x}{(x-a)^p};$$

(2)
$$\int_0^1 \frac{dx}{1-x^2}$$
;

双数? 若收敛,则求(2) \(\int_0 \) \(\frac{dx}{1-x^2} \); WHITHOUN, 31 ALSO SALOS , WHITH HIS HITH HIS AND SALOS , WHITH HIS AND SALOS S



(5) John zdz; (6) John zdz; Willith 8820588 Allik 826633150° talling the high of the same of t White in the state of the state



3. 举例说明: 瑕积分 $\int_{a}^{b} f(z) dz$ 收敛时, $\int_{a}^{b} f(z) dz$ 不一定收敛.

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4. 举例说明: $\int_{a}^{\infty} f(z) dz$ 收敛且 f 在 $[a, +\infty)$ 上连续时,不一定有 $\lim_{z\to +\infty} f(z) = 0$.

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5. 证明:若 $\int_{a}^{\infty} f(z) dz$ 收敛,且存在极限 $\lim_{z\to\infty} f(z) = A$,则 A = 0.

William States of the second o 6. 证明:著/在[a, + m)上可导,且[**f(a)da.与[**f'(a)da.物收敛,则 lim/f(a) = 0.



1. 证明定理 11.2 及其推论 1.



2. 设f与g是定义在 $[a,+\infty)$ 上的函数,对任何u>a,它们在[a,u]上都可积. 证明:若 $\int_{-\infty}^{\infty} f(x) dx$ 与 $\int_{-\infty}^{\infty} g^2(x) dx$ 收敛,则 $\int_{-\infty}^{\infty} f(x) g(x) dx$ 与 $\int_{-\infty}^{\infty} [f(x) + g(x)]^2 dx$ 也都收敛.

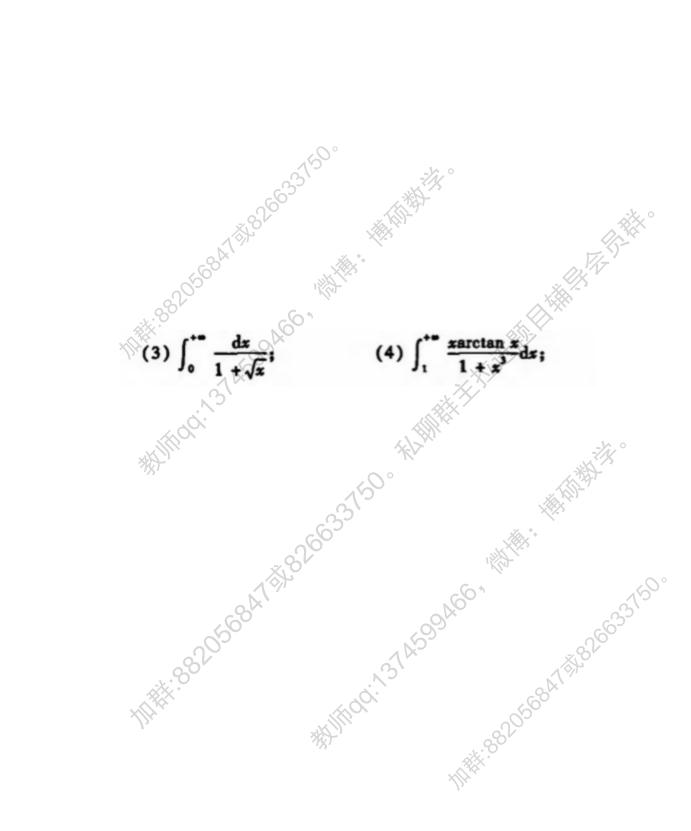
(1) 若
$$\int_{a}^{\infty} h(x) dx$$
与 $\int_{a}^{\infty} g(x) dx$ 都收敛,则 $\int_{a}^{\infty} f(x) dx$ 也收敛;
(2) 又若 $\int_{a}^{\infty} h(x) dx = \int_{a}^{\infty} g(x) dx = A$,则 $\int_{a}^{\infty} f(x) dx = A$.

(2) 又若
$$\int_{0}^{\infty} h(x) dx = \int_{0}^{\infty} g(x) dx = A$$
 ,则 $\int_{0}^{\infty} f(x) dx = A$.

NOTAL SOME 4. 讨论下列无穷积分的收敛性:

$$(1) \int_0^{\infty} \frac{dx}{\sqrt[3]{x^4 + 1}};$$

$$(2) \int_{1}^{\infty} \frac{x}{1 - e^{x}} dx_{1}$$



 $(5) \int_{0}^{+\infty} \frac{\ln(1+x)}{x^{n}} dx; \qquad (6) \int_{0}^{+\infty} \frac{x^{n}}{1+x^{n}} dx(n,m \ge 0).$ MIRE SOURCE SALES SOURCE STATE OF SOURCE SALES SALES SOURCE SALES SALES SOURCE SALES SALE Whilliadi. 31 Ato 99 Arob 3 Arbon 1914 Arith in the state of the state

5. 讨论下列无穷积分为绝对收敛还是条件收敛:

(1)
$$\int_{1}^{\infty} \frac{\sin\sqrt{x}}{x} dx;$$
 (2)
$$\int_{0}^{\infty} \frac{\operatorname{sgn}(\sin x)}{1+x^{2}} dx;$$

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(3)
$$\int_0^{\infty} \frac{\sqrt{x \cos x}}{100 + x} dx;$$
 (4)
$$\int_0^{\infty} \frac{\ln(\ln x)}{\ln x} \sin x dx.$$

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6. 举例说明: $\int_a^b f(x) dx$ 收敛时 $\int_a^b f'(x) dx$ 不一定收敛; $\int_a^b f(x) dx$ 绝对收敛时, $\int_{0}^{\infty} f^{2}(x) dx$ 也不一定收敛.

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7. 证明:若 $\int_{a}^{\infty} f(x) dx$ 绝对收敛,且 $\lim_{x\to +\infty} f(x) = 0$,则 $\int_{a}^{\infty} f'(x) dx$ 必定收敛.

8. 证明:若f是 $[a, +\infty)$ 上的单调函数,且 $\int_{-\infty}^{+\infty} f(x) dx$ 收敛,则 $\lim_{x \to +\infty} f(x) = 0$,且f(x) = 0White in the state of the state 9. 证明:若f在 $[a, +\infty)$ 上一致连续,且 $\int_{a}^{\infty} f(x) dx$ 收敛,则 $\lim_{x\to +\infty} f(x) = 0$.

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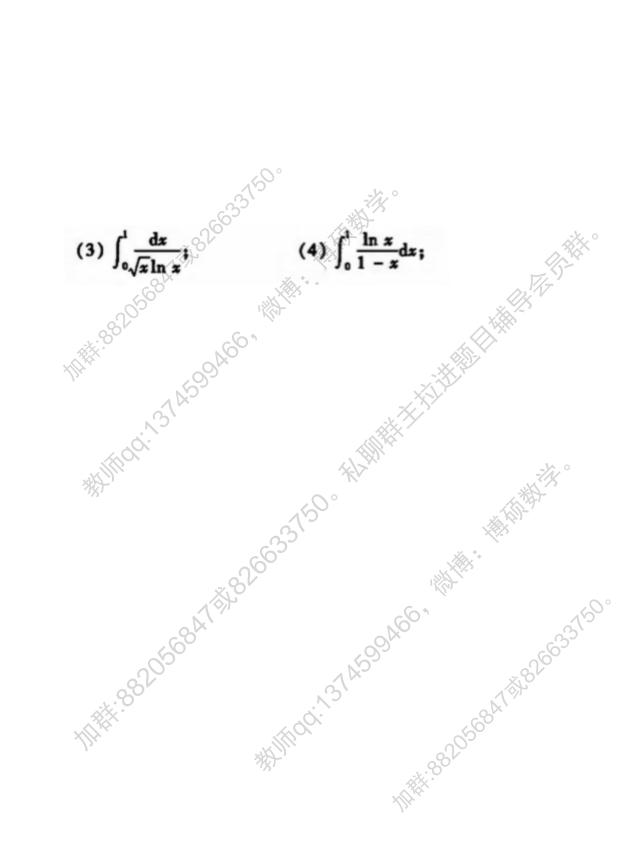
第 11.3 节 瑕积分的性质与收敛判别

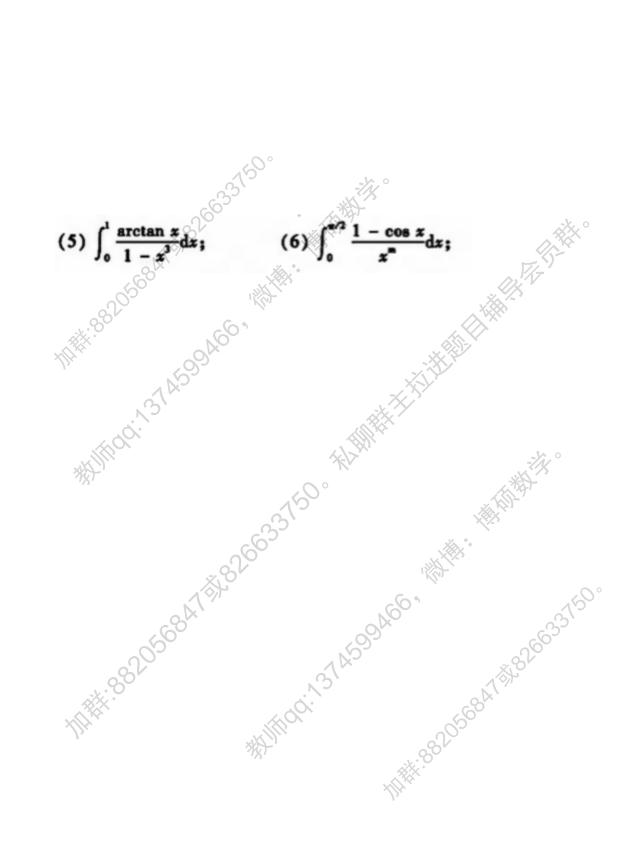


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(1)
$$\int_0^2 \frac{dx}{(x-1)^2}$$
; (2) $\int_0^x \frac{\sin x}{x^{3/2}} dx$

1)^{2;} (2) \(\frac{\sin x}{x^{3/2}} \, \dx; \) WHITHOUT STATES SAGES , WHITH HITH THE CONTROL OF T





(7)
$$\int_0^1 \frac{1}{x^{\alpha}} \sin \frac{1}{x} dx$$
; (8) $\int_0^{+\infty} e^{-x} \ln x dx$.



(1)
$$\int_0^1 (\ln x)^n dx$$
; (2) $\int_0^1 \frac{x^n}{\sqrt{1-x}} dx$.

计算下列现积分的(1) ∫ (ln z) 'dz; 0° With 8826833150° Killing Hill Hill Hill Back Salah Sal White of 1914 of 1914

5. 证明瑕积分
$$J = \int_0^{\pi/2} \ln(\sin x) dx$$
 收敛,且 $J = -\frac{\pi}{2} \ln 2$.(提示:利用 $\int_0^{\pi/2} \ln(\cos x) dx$,并将它们相加.)

6. 利用上题结果,证明:

$$(1) \int_0^{\pi} \theta \ln(\sin \theta) d\theta = -\frac{\pi^2}{2} \ln 2;$$

AND STATE OF A STATE O $(2) \int_0^{\pi} \frac{\theta \sin \theta}{1 - \cos \theta} d\theta = 2\pi \ln 2.$