量子力学常用积分公式

(1) $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx \qquad (n > 0)$

(2) $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$

(3) $\int e^{ax} \cos ax dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$

(4) $\int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{1}{a} x \cos ax$

(5) $\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + (\frac{2}{a^2} - \frac{x^2}{a}) \cos ax$

(6) $\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$

(7 $\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + (\frac{x^2}{a} - \frac{2}{a^3}) \sin ax$)

(8) $\int \sqrt{ax^2 + c} dx =$

$$\begin{cases} \frac{x}{2}\sqrt{ax^2 + c} + \frac{c}{2\sqrt{a}} \ln(\sqrt{a}x + \sqrt{ax^2 + c}) & (a > 0) \\ \frac{x}{2}\sqrt{ax^2 + c} + \frac{c}{2\sqrt{-a}} \arcsin(\sqrt{\frac{-a}{c}}x) & (a<0) \end{cases}$$

(9) $\int_0^{\frac{\pi}{2}} \sin^n x dx$ $\begin{cases} \frac{(n-1)!!}{n!!}\frac{\pi}{2} & (n = \text{正偶数}) \\ \frac{(n-1)!!}{n!!} & (n = \text{正奇数}) \end{cases}$ = $\int_0^{\frac{\pi}{2}} \cos^n x dx$

$$\begin{cases} \frac{\pi}{2} & (a > 0) \\ \end{cases}$$

(10) $\int_0^\infty \frac{\sin ax}{x} dx =$

$-\frac{\pi}{2}$ $(a<0)$

(11)) $\int_0^\infty e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$ $(n=$正整数$, a>0)$

(12) $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

(13) $\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$

(14) $\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$

(15) $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$

(16) $\int_0^\infty xe^{-ax} \sin bx dx = \frac{2ab}{(a^2+b^2)^2}$ $(a>0)$

$\int_0^\infty xe^{-ax} \cos bx dx = \frac{a^2-b^2}{(a^2+b^2)^2}$ $(a>0)$