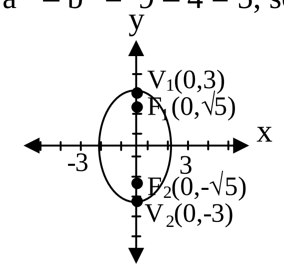
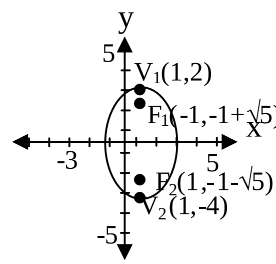
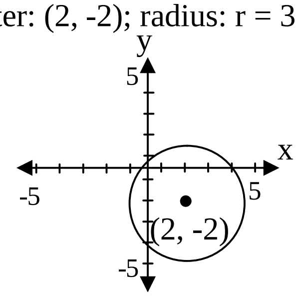
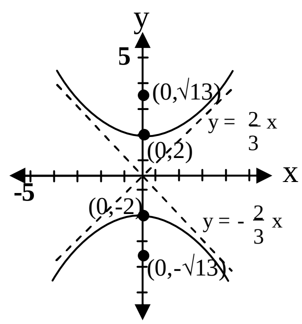
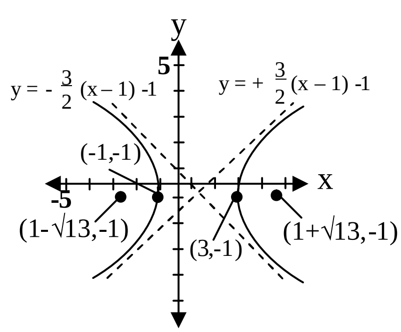
ELLIPSE, HYPERBOLA AND PARABOLA

ELLIPSE

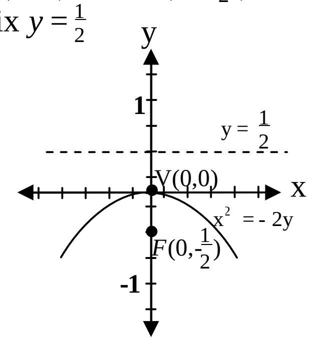
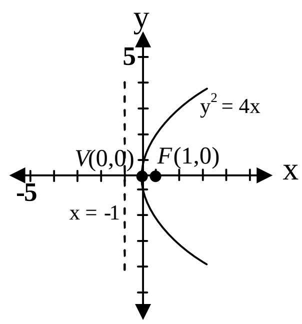
Concept	Equation	Example
Ellipse with Center (0, 0)	Standard equation with a > b > 0 Horizontal major axis: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Vertical major axis $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	$\frac{x^2}{4} + \frac{y^2}{9} = 1$; a = 3. b = 2 Center (0, 0); major axis: vertical Vertices: (0, ±3); foci: (0, ± √5) ($c^2 = a^2 - b^2 = 9 - 4 = 5$, so c = √5.) 
Ellipse with center (h, k)	Standard equation with a > b > 0 Horizontal major axis: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Vertical major axis $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$; a = 3. b = 2 
Circle with center (h, k) and radius r	Standard equation $(x-h)^2 + (y-k)^2 = r^2$ A circle is an ellipse with a = b = r.	$(x-2)^2 + (y+2)^2 = 9$ Center: (2, -2); radius: r = 3 
Area inside an ellipse	$A = \pi ab$	The area inside the ellipse give by $\frac{x^2}{49} + \frac{y^2}{9} = 1$ is $A = \pi(7)(3) = 21\pi$ square units.

HYPERBOLA

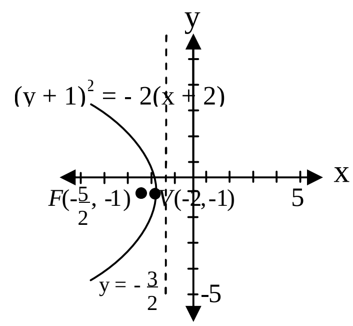
Concept	Equation	Example
Hyperbola with center (0, 0)	Standard equation Transverse axis: horizontal $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Transverse axis: vertical $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{x^2}{4} - \frac{y^2}{9} = 1$; a = 2. b = 3 Transverse axis: vertical Vertices (0, ± 2); foci: (0, ±√13) ($c^2 = a^2 + b^2 = 4 + 9 = 13$, so c = √13.) Asymptotes: $y = \pm \frac{2}{3} x$ 
Hyperbola with center (h, k)	Standard Equation Transverse axis: horizontal $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Transverse axis: vertical $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$\frac{(x-1)^2}{4} - \frac{(y+1)^2}{9} = 1$; a = 2. b = 3 Transverse axis: horizontal; center (1, -1) Vertices (1 ± 2, - 1); foci: (1 ± √13, - 1) ($c^2 = a^2 + b^2 = 4 + 9 = 13$, so c = √13.) Asymptotes: $y = \pm \frac{3}{2} (x - 1) - 1$ 

PARABOLAS

Parabola Vertex (0, 0)

Concept	Equation	Example
Parabola with vertex (0, 0) and vertical axis	$x^2 = 4py$ $p > 0$: opens upward $p < 0$: opens downward Focus: $(0, p)$ Directrix: $y = -p$	$x^2 = -2y$ has $4p = -2$ or $p = -\frac{1}{2}$ The parabola opens downward with vertex (0, 0), focus $(0, -\frac{1}{2})$, and directrix $y = \frac{1}{2}$ 
Parabola with vertex (0, 0) and horizontal axis	$y^2 = 4px$ $p > 0$: opens to the right $p < 0$: opens to the left Focus: $(p, 0)$ Directrix: $x = -p$	$y^2 = 4x$ has $4p = 4$ or $p = 1$ The parabola opens to he right with vertex (0, 0), focus (1, 0), and directrix $x = -1$ 

Parabola Vertex (h, k)

Concept	Equation	Example
Parabola with vertex (h, k) and horizontal axis	$(y - k)^2 = 4p(x - h)$ p > 0: opens to the right p < 0: opens to the left Focus: (h + p, k) Directrix: x = h - p	$(y + 1)^2 = -2(x + 2)$ has $p = -\frac{1}{2}$ The parabola opens to the left with vertex (-2, - 1), focus ($-\frac{5}{2}$, -1), and directrix x = $-\frac{3}{2}$ 
Parabola with vertex (h, k) and vertical axis	$(x - h)^2 = 4p(y - k)$ p > 0: opens upwards p < 0: opens downwards Focus: (h, k + p) Directrix: y = k - p	$(x - 1)^2 = 8(y - 3)$ has $p = 2$. The parabola opens upward with vertex (1, 3), focus (1, 5), and directrix y = 1.