附录II　初等数学常用公式

一、代数

1、绝对值

(1) 定义：$|a|=\begin{cases}a, a\geq 0,\\ -a, a<0.\end{cases}$

(2) 性质：$|a|=|-a|$，$|ab|=|a|\cdot|b|$，$\left|\frac{a}{b}\right|=\frac{|a|}{|b|}(b\neq 0)$，

$|a|\leq A\Rightarrow -A\leq a\leq A$，

$|a|\geq A\Leftrightarrow a\geq A$ 或 $a\leq -A$

$|a|-|b|\leq|a\pm b|\leq|a|+|b|$.

2、指数

(1) $a^m\cdot a^n=a^{m+n}$　　(2) $\frac{a^m}{a^n}=a^{m-n}$　　(3) $(a^m)^n=a^{mn}$

(4) $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$　　(5) $(ab)^m=a^mb^m$ (a，b是正实数，m，n是任意实数)

3、对数

设 a > 0, a ≠ 1, 则

(1) $\log_a xy = \log_a x + \log_a y$　　(2) $\log_a \frac{x}{y} = \log_a x - \log_a y$

(3) $\log_a x^b = b\log_a x$　　(4) $\log_a x = \frac{\log_b x}{\log_b a}$

(5) $a^{\log_a x} = x, \log_a 1 = 0, \log_a a = 1$

4、二项展开与分解公式

(1) $(a\pm b)^2=a^2\pm 2ab+b^2$

(2) $(a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$

(3) $a^2-b^2=(a+b)(a-b)$

(4) $a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$

(5) $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\cdots+ab^{n-2}+b^{n-1})$

(6)

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdots + \frac{n(n-1)\cdots(n-k+1)}{k!}a^{n-k}b^k + \cdots + b^n$$

5、数列

$$(1)\ a + aq + aq^2 + \cdots + aq^{n-1} = \frac{a(1-q^n)}{1-q},\ |q| \neq 1$$

$$(2)\ 1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$$

$$(3)\ 1 + 3 + 5 + \cdots + (2n-1) = n^2$$

$$(4)\ 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$(5)\ 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

二、三角

1. 基本公式

(1) $\sin^2\alpha+\cos^2\alpha=1$　(2) $1+\tan^2\alpha=\sec^2\alpha$　(3) $1+\cot^2\alpha=\csc^2\alpha$　(4) $\frac{\sin\alpha}{\cos\alpha}=\tan\alpha$

(5) $\frac{\cos\alpha}{\sin\alpha}=\cot\alpha$　(6) $\cot\alpha=\frac{1}{\tan\alpha}$　(7) $\csc\alpha=\frac{1}{\sin\alpha}$　(8) $\sec\alpha=\frac{1}{\cos\alpha}$

2. 和差公式

(1) $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$　(2) $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

(3) $\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$　(4) $\cot(\alpha\pm\beta)=\frac{\cot\alpha\cot\beta\mp1}{\cot\beta\pm\cot\alpha}$

3. 倍角和半角公式

(1) $\sin2\alpha=2\sin\alpha\cos\alpha$

(2) $\cos2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1$

(3) $\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$

(4) $\cot2\alpha=\frac{\cot^2\alpha-1}{2\cot\alpha}$

(5) $\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$

(6) $\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$

(7) $\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}$

(8) $\cot\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$

4. 和差化积公式

(1) $\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$　　(2) $\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

(3) $\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$　　(4) $\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

5. 积化和差公式

(1) $\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$

(2) $\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$

(3) $\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]$

三、几何

1. 平面图形的基本公式

(1) 梯形面积 $S=\frac{1}{2}(a+b)h$，(其中 a，b 为二底，h 为高).

(2) 圆面积 $S=\pi R^2$，圆周长 $l=2\pi R$，(R 是圆半径).

(3) 圆扇形面积 $S=\frac{1}{2}R^2\theta$，圆扇形弧长 $l=R\theta$，(R 是圆半径，θ 为圆心角，单位为弧度)

2. 立体图形的基本公式

(1) 圆柱体体积 $V=\pi R^2H$，圆柱体侧面积 $S=2\pi RH$，(其中 R 是底半径，H 是高)

(2) 正圆锥体体积 $V=\frac{1}{3}\pi R^2H$，侧面积 $S=\pi Rl$，(其中 l 为斜高，即 $l=\sqrt{R^2+H^2}$).

(3) 棱柱体积 $V=SH$，(S 为底面积，H 为高).

(4) 棱锥体体积 $V=\frac{1}{3}SH$，(S 为底面积，H 为高).

(5) 球体积 $V=\frac{4}{3}\pi R^3$.

(6) 球面积 $S=4\pi R^2$，(R 为球的半径).

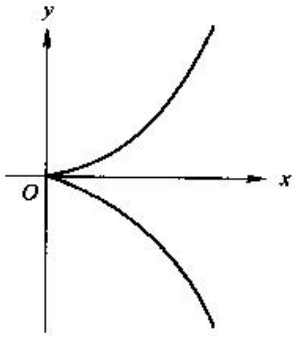
(7)圆台

体积 $V=\frac{1}{3}\pi h(R^2+Rr+r^2)$，侧面积 $S=\pi l(R+r)$，R 与 r 分别为上下底半径，h 为高，l 为斜高.

附录III　常用曲线

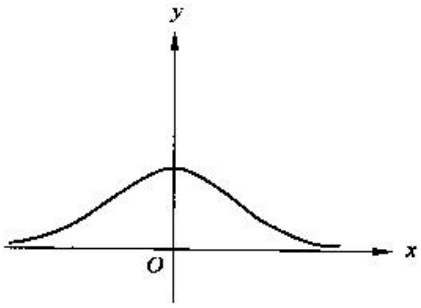
1. 半立方抛物线

$y^2=ax^3$



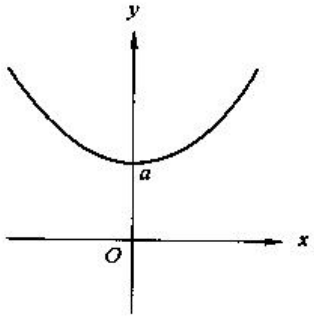
2. 概率曲线

$y=e^{-\frac{x^2}{2}}$



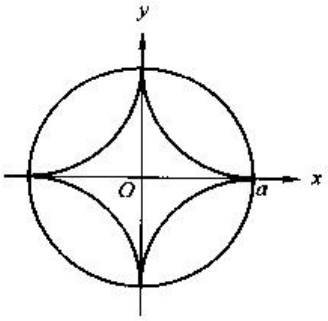
3. 悬链线

$y=a\cosh\frac{x}{a}$



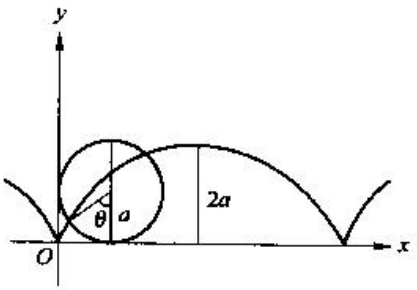
4. 星形线(内摆线)

$x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ 或 $\begin{cases}x=a\cos^3\theta\\y=a\sin^3\theta\end{cases}$



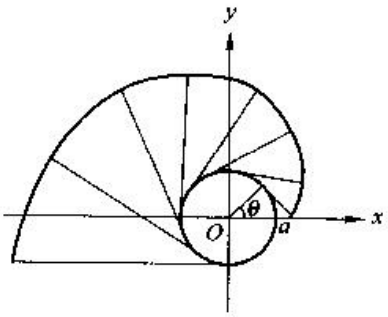
5. 摆线

$\begin{cases}x=a(\theta-\sin\theta)\\y=a(1-\cos\theta)\end{cases}$



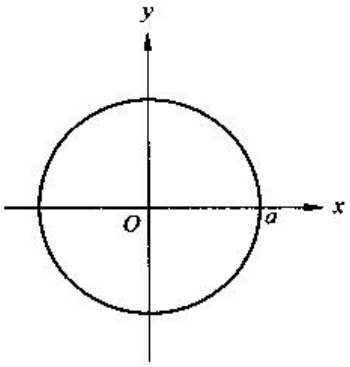
6. 圆的渐开线

$\begin{cases}x=a(\cos\theta+\theta\sin\theta)\\y=a(\sin\theta-\theta\cos\theta)\end{cases}$



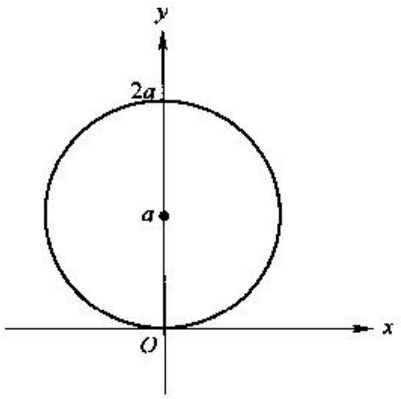
7. 圆

$x^2+y^2=a^2$ 或 $r=a$



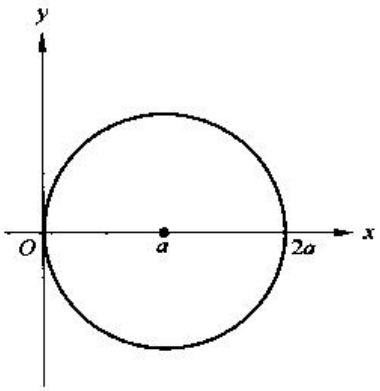
8. 圆

$x^2+(y-a)^2=a^2$ 或 $r=2a\sin\theta$



9. 圆

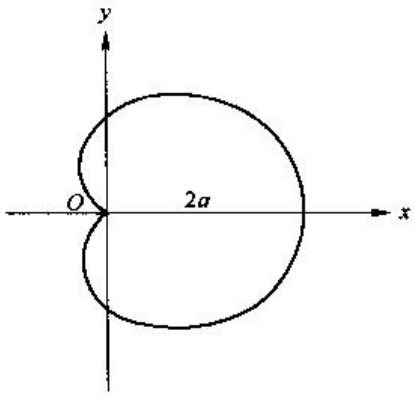
$(x-a)^2+y^2=a^2$ 或 $r=2a\cos\theta$



10. 心形线

$x^2+y^2-ax=a\sqrt{x^2+y^2}$

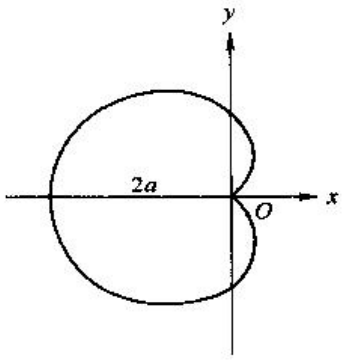
或 $r=a(1+\cos\theta)$



11. 心形线

$x^2+y^2+ax=a\sqrt{x^2+y^2}$

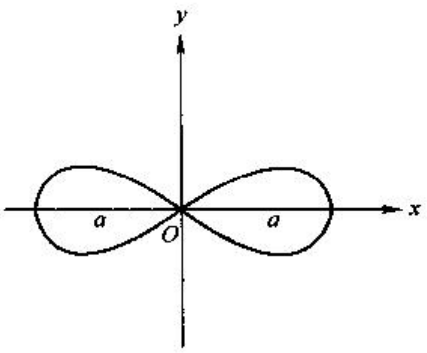
或 $r=a(1-\cos\theta)$



12. 双纽线

$(x^2+y^2)^2=a^2(x^2-y^2)$

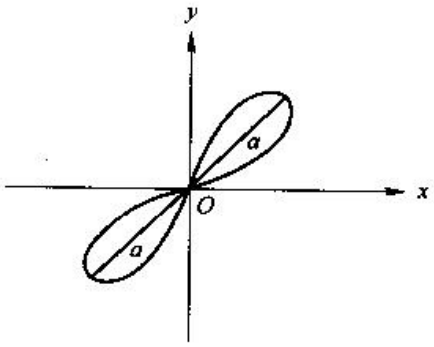
或 $r^2=a^2\cos 2\theta$



13. 双纽线

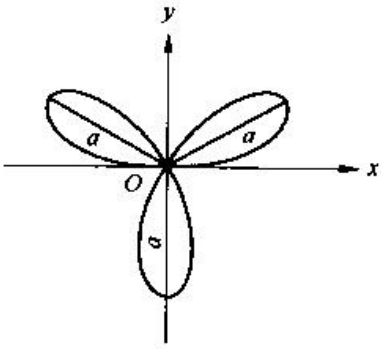
$(x^2+y^2)^2=2a^2xy$

或 $r^2=a^2\sin 2\theta$



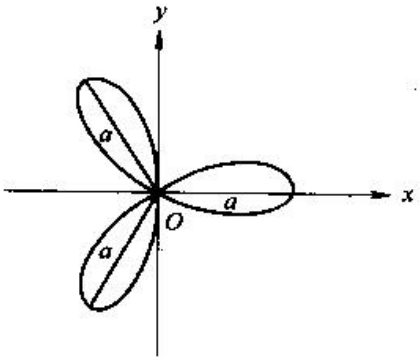
14. 三叶玫瑰线

$r=a\sin 3\theta$



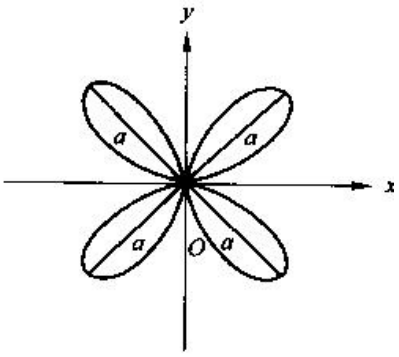
15. 三叶玫瑰线

$r=a\cos 3\theta$



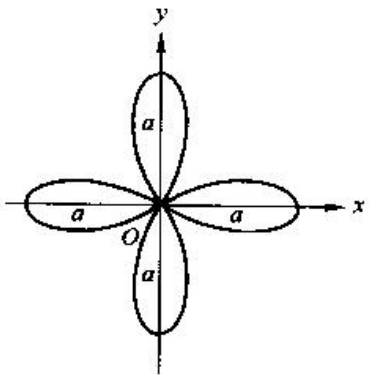
16. 四叶玫瑰线

$r=a\sin 2\theta$



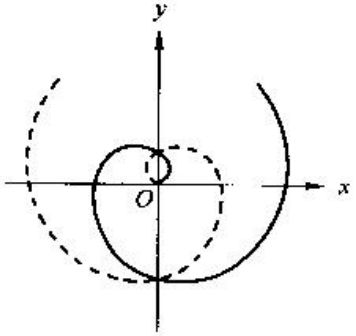
17. 四叶玫瑰线

$r=a\cos 2\theta$



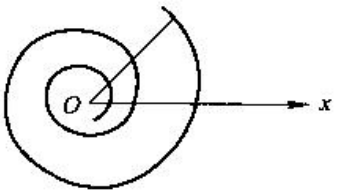
18. 阿基米德螺线

$r=a\theta$



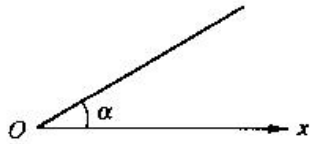
19. 对数螺线

$r=e^{a\theta}$



20. 射线

$\theta=\alpha$



附录Ⅳ 积分表

（一）含有 $a+bx$ 的积分

1. $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)+C$

2. $\int (a+bx)^{\mu}dx = \frac{(a+bx)^{\mu+1}}{b(\mu+1)}+C \quad (\mu \neq -1)$

3. $\int \frac{xdx}{a+bx} = \frac{1}{b^2}[a+bx-a\ln(a+bx)]+C$

4. $\int \frac{x^2dx}{a+bx} = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2-2a(a+bx)+a^2\ln(a+bx)\right]+C$

5. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\ln\frac{a+bx}{x}+C$

6. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax}+\frac{b}{a^2}\ln\frac{a+bx}{x}+C$

7. $\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left[\ln(a+bx)+\frac{a}{a+bx}\right]+C$

8. $\int \frac{x^2dx}{(a+bx)^2} = \frac{1}{b^3}\left[a+bx-2a\ln(a+bx)-\frac{a^2}{a+bx}\right]+C$

9. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)}-\frac{1}{a^2}\ln\frac{a+bx}{x}+C$

（二）含有$\sqrt{a+bx}$的积分

10. $\int \sqrt{a+bx}dx = \frac{2}{3b}\sqrt{(a+bx)^3}+C$

11. $\int x\sqrt{a+bx}dx = -\frac{2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2}+C$

12. $\int x^2\sqrt{a+bx}dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3}+C$

13. $\int \frac{xdx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx}+C$

14. $\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx}+C$

15. $\int \frac{dx}{x\sqrt{a+bx}} = \begin{cases} \frac{1}{\sqrt{a}}\ln\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}+C & (a>0) \\ \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bx}{-a}}+C & (a<0) \end{cases}$

16. $\int \frac{\mathrm{d}x}{x^2\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}\int \frac{\mathrm{d}x}{x\sqrt{a+bx}}$

17. $\int \frac{\sqrt{a+bx}\,\mathrm{d}x}{x} = 2\sqrt{a+bx} + a\int \frac{\mathrm{d}x}{x\sqrt{a+bx}}$

（三）含有 $a^2 \pm x^2$ 的积分

18. $\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

19. $\int \frac{\mathrm{d}x}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2}\int \frac{\mathrm{d}x}{(x^2+a^2)^{n-1}}$

20. $\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x} + C \quad (|x|<a)$

21. $\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\frac{x-a}{x+a} + C \quad (|x|>a)$

（四）含有 $a \pm bx^2$ 的积分

22. $\int \frac{\mathrm{d}x}{a+bx^2} = \frac{1}{\sqrt{ab}}\arctan\sqrt{\frac{b}{a}}x + C \quad (a>0, b>0)$

23. $\int \frac{\mathrm{d}x}{a-bx^2} = \frac{1}{2\sqrt{ab}}\ln\frac{\sqrt{a}+\sqrt{b}x}{\sqrt{a}-\sqrt{b}x} + C$

24. $\int \frac{x\mathrm{d}x}{a+bx^2} = \frac{1}{2b}\ln(a+bx^2) + C$

25. $\int \frac{x^2\mathrm{d}x}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{\mathrm{d}x}{a+bx^2}$

26. $\int \frac{\mathrm{d}x}{x(a+bx^2)} = \frac{1}{2a}\ln\frac{x^2}{a+bx^2} + C$

27. $\int \frac{\mathrm{d}x}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{\mathrm{d}x}{a+bx^2}$

28. $\int \frac{\mathrm{d}x}{(a+bx^2)^2} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a}\int \frac{\mathrm{d}x}{a+bx^2}$

（五）含有 $\sqrt{x^2+a^2}$ 的积分

29. $\int \sqrt{x^2+a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$

30. $\int \sqrt{(x^2+a^2)^3}\,\mathrm{d}x = \frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2} + \frac{3a^4}{8}\ln(x+\sqrt{x^2+a^2}) + C$

31. $\int x\sqrt{x^2+a^2}\,\mathrm{d}x = \frac{\sqrt{(x^2+a^2)^3}}{3} + C$

32. $\int x^2\sqrt{x^2+a^2}\,\mathrm{d}x = \frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \frac{a^4}{8}\ln(x+\sqrt{x^2+a^2}) + C$

33. $\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) + C$

34. $\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$

35. $\int \frac{xdx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$

36. $\int \frac{x^2dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$

37. $\int \frac{x^2dx}{\sqrt{(x^2+a^2)^3}} = -\frac{x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2}) + C$

38. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\frac{x}{a+\sqrt{x^2+a^2}} + C$

39. $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x} + C$

40. $\int \frac{\sqrt{x^2+a^2}dx}{x} = \sqrt{x^2+a^2} - a\ln\frac{a+\sqrt{x^2+a^2}}{x} + C$

41. $\int \frac{\sqrt{x^2+a^2}dx}{x^2} = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2}) + C$

（六）含有$\sqrt{x^2-a^2}$的积分

42. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2}) + C_1 = \operatorname{arcosh}\frac{x}{a} + C$

43. $\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$

44. $\int \frac{xdx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + C$

45. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2}) + C$

46. $\int \sqrt{(x^2-a^2)^3}dx = \frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \frac{3a^4}{8}\ln(x+\sqrt{x^2-a^2}) + C$

47. $\int x\sqrt{x^2-a^2}dx = \frac{\sqrt{(x^2-a^2)^3}}{3} + C$

48. $\int x\sqrt{(x^2-a^2)^3}dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + C$

49. $\int x^2\sqrt{x^2-a^2}dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln(x+\sqrt{x^2-a^2}) + C$

50. $\int \frac{x^2dx}{\sqrt{x^2-a^2}} = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2}) + C$

51. $\int \frac{x^2dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{\sqrt{x^2-a^2}} + \ln(x+\sqrt{x^2-a^2}) + C$

52. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\arccos\frac{a}{x} + C$

53. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$

54. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\arccos\frac{a}{x} + C$

55. $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x+\sqrt{x^2-a^2}) + C$

(七) 含有$\sqrt{a^2-x^2}$的积分

56. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

57. $\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}} + C$

58. $\int \frac{xdx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2} + C$

59. $\int \frac{xdx}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}} + C$

60. $\int \frac{x^2dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

61. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

62. $\int \sqrt{(a^2-x^2)^3}dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\arcsin\frac{x}{a} + C$

63. $\int x\sqrt{a^2-x^2}dx = -\frac{\sqrt{(a^2-x^2)^3}}{3} + C$

64. $\int x\sqrt{(a^2-x^2)^3}dx = -\frac{\sqrt{(a^2-x^2)^5}}{5} + C$

65. $\int x^2\sqrt{a^2-x^2}dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\arcsin\frac{x}{a} + C$

66. $\int \frac{x^2dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{\sqrt{a^2-x^2}} - \arcsin\frac{x}{a} + C$

67. $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\ln\frac{x}{a+\sqrt{a^2-x^2}} + C$

68. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

69. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\frac{a+\sqrt{a^2-x^2}}{x} + C$

70. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\frac{\sqrt{a^2-x^2}}{x} - \arcsin\frac{x}{a} + C$

（八）含有 $a+bx\pm cx^2\ (c>0)$ 的积分

71. $\int \frac{dx}{a+bx-cx^2}=\frac{1}{\sqrt{b^2+4ac}}\ln\frac{\sqrt{b^2+4ac}+2cx-b}{\sqrt{b^2+4ac}-2cx+b}+C$

72. $\int \frac{dx}{a+bx+cx^2}=\begin{cases}\frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2cx+b}{\sqrt{4ac-b^2}}+C & (b^2<4ac)\\ \frac{1}{\sqrt{b^2-4ac}}\ln\frac{2cx+b-\sqrt{b^2-4ac}}{2cx+b+\sqrt{b^2-4ac}}+C & (b^2>4ac)\end{cases}$

（九）含有 $\sqrt{a+bx\pm cx^2}\ (c>0)$ 的积分

73. $\int \frac{dx}{\sqrt{a+bx+cx^2}}=\frac{1}{\sqrt{c}}\ln\left(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}\right)+C$

74. $\int \sqrt{a+bx+cx^2}dx=\frac{2cx+b}{4c}\sqrt{a+bx+cx^2}-\frac{b^2-4ac}{8\sqrt{c^3}}\ln(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2})+C$

75. $\int \frac{xdx}{\sqrt{a+bx+cx^2}}=\frac{\sqrt{a+bx+cx^2}}{c}-\frac{b}{2\sqrt{c^3}}\ln\left(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}\right)+C$

76. $\int \frac{dx}{\sqrt{a+bx-cx^2}}=\frac{1}{\sqrt{c}}\arcsin\frac{2cx-b}{\sqrt{b^2+4ac}}+C$

77. $\int \sqrt{a+bx-cx^2}dx=\frac{2cx-b}{4c}\sqrt{a+bx-cx^2}+\frac{b^2+4ac}{8\sqrt{c^3}}\arcsin\frac{2cx-b}{\sqrt{b^2+4ac}}+C$

78. $\int \frac{xdx}{\sqrt{a+bx-cx^2}}=-\frac{\sqrt{a+bx-cx^2}}{c}+\frac{b}{2\sqrt{c^3}}\arcsin\frac{2cx-b}{\sqrt{b^2+4ac}}+C$

（十）含有 $\sqrt{\frac{a\pm x}{b\pm x}}$ 和 $\sqrt{(x-a)(b-x)}$ 的积分

79. $\int\sqrt{\frac{a+x}{b+x}}dx=\sqrt{(a+x)(b+x)}+(a-b)\ln\left(\sqrt{a+x}+\sqrt{b+x}\right)+C$

80. $\int\sqrt{\frac{a-x}{b+x}}dx=\sqrt{(a-x)(b+x)}+(a+b)\arcsin\sqrt{\frac{x+b}{a+b}}+C$

81. $\int\sqrt{\frac{a+x}{b-x}}dx=-\sqrt{(a+x)(b-x)}-(a+b)\arcsin\sqrt{\frac{b-x}{a+b}}+C$

82. $\int \frac{dx}{\sqrt{(x-a)(b-x)}}=2\arcsin\sqrt{\frac{x-a}{b-a}}+C$

（十一）含有三角函数的积分

83. $\int \sin x dx = -\cos x + C$

84. $\int \cos x dx = \sin x + C$

85. $\int \tan x dx = -\ln \cos x + C$

86. $\int \cot x dx = \ln \sin x + C$

87. $\int \sec x dx = \ln (\sec x + \tan x) + C = \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$

88. $\int \csc x dx = \ln(\csc x - \cot x) + C = \ln \tan \frac{x}{2} + C$

89. $\int \sec^2 x dx = \tan x + C$

90. $\int \csc^2 x dx = -\cot x + C$

91. $\int \sec x \tan x dx = \sec x + C$

92. $\int \csc x \cot x dx = -\csc x + C$

93. $\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

94. $\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$

95. $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx$

96. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx$

97. $\int \frac{dx}{\sin^n x} = -\frac{1}{n-1}\frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x}$

98. $\int \frac{dx}{\cos^n x} = \frac{1}{n-1}\frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x}$

99. $\int \cos^m x \sin^n x dx = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \cos^{m-2} x \sin^n x dx$

$$= -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2} x dx$$

100. $\int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C \quad (m \neq n)$

101. $\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C \quad (m \neq n)$

102. $\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C \quad (m \neq n)$

103. $\int \frac{dx}{a + b\sin x} = \frac{2}{a\sqrt{\frac{a^2}{a^2 - b^2}}} \arctan\left[\sqrt{\frac{a^2}{a^2 - b^2}}\left(\tan \frac{x}{2} + \frac{b}{a}\right)\right] + C \quad (a^2 > b^2)$

104. $\int \frac{dx}{a + b\sin x} = \frac{1}{a\sqrt{\frac{a^2}{b^2 - a^2}}} \ln \frac{\tan \frac{x}{2} + \frac{b}{a} - \sqrt{\frac{b^2 - a^2}{a^2}}}{\tan \frac{x}{2} + \frac{b}{a} + \sqrt{\frac{b^2 - a^2}{a^2}}} + C \quad (a^2 < b^2)$

105. $\int \frac{dx}{a + b\cos x} = \frac{2}{a - b}\sqrt{\frac{a-b}{a+b}} \arctan\left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2}\right) + C \quad (a^2 > b^2)$

106. $\int \frac{dx}{a + b\cos x} = \frac{1}{b - a}\sqrt{\frac{b-a}{b+a}} \ln \frac{\tan \frac{x}{2} + \sqrt{\frac{b+a}{b-a}}}{\tan \frac{x}{2} - \sqrt{\frac{b+a}{b-a}}} + C \quad (a^2 < b^2)$

107. $\int \frac{\mathrm{d}x}{a^2\cos^2 x + b^2\sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b\tan x}{a}\right) + C$

108. $\int \frac{\mathrm{d}x}{a^2\cos^2 x - b^2\sin^2 x} = \frac{1}{2ab}\ln\frac{b\tan x + a}{b\tan x - a} + C$

109. $\int x\sin ax\,\mathrm{d}x = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$

110. $\int x^2\sin ax\,\mathrm{d}x = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C$

111. $\int x\cos ax\,\mathrm{d}x = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$

112. $\int x^2\cos ax\,\mathrm{d}x = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C$

（十二）含有反三角函数的积分

113. $\int \arcsin\frac{x}{a}\mathrm{d}x = x\arcsin\frac{x}{a} + \sqrt{a^2 - x^2} + C$

114. $\int x\arcsin\frac{x}{a}\mathrm{d}x = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{a^2 - x^2} + C$

115. $\int x^2\arcsin\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2 + 2a^2)\sqrt{a^2 - x^2} + C$

116. $\int \arccos\frac{x}{a}\mathrm{d}x = x\arccos\frac{x}{a} - \sqrt{a^2 - x^2} + C$

117. $\int x\arccos\frac{x}{a}\mathrm{d}x = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2 - x^2}$

118. $\int x^2\arccos\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2 + 2a^2)\sqrt{a^2 - x^2} + C$

119. $\int \arctan\frac{x}{a}\mathrm{d}x = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2 + x^2) + C$

120. $\int x\arctan\frac{x}{a}\mathrm{d}x = \frac{1}{2}(x^2 + a^2)\arctan\frac{x}{a} - \frac{ax}{2} + C$

121. $\int x^2\arctan\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arctan\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(a^2 + x^2) + C$

（十三）含有指数函数的积分

122. $\int a^x\mathrm{d}x = \frac{a^x}{\ln a} + C$

123. $\int \mathrm{e}^{ax}\mathrm{d}x = \frac{\mathrm{e}^{ax}}{a} + C$

124. $\int \mathrm{e}^{ax}\sin bx\,\mathrm{d}x = \frac{\mathrm{e}^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2} + C$

125. $\int \mathrm{e}^{ax}\cos bx\,\mathrm{d}x = \frac{\mathrm{e}^{ax}(b\sin bx + a\cos bx)}{a^2 + b^2} + C$

126. $\int x\mathrm{e}^{ax}\mathrm{d}x = \frac{\mathrm{e}^{ax}}{a^2}(ax - 1) + C$

127. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax}\,dx$

128. $\int x a^{mx}\,dx = \frac{x a^{mx}}{m\ln a} - \frac{a^{mx}}{(m\ln a)^2} + C$

129. $\int x^n a^{mx}\,dx = \frac{a^{mx} x^n}{m\ln a} - \frac{n}{m\ln a}\int x^{n-1} a^{mx}\,dx$

130. $\int e^{ax}\sin^n bx\,dx = \frac{e^{ax}\sin^{n-1} bx}{a^2+b^2n^2}(a\sin bx - nb\cos bx) + \frac{n(n-1)}{a^2+b^2n^2}b^2\int e^{ax}\sin^{n-2} bx\,dx$

131. $\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1} bx}{a^2+b^2n^2}(a\cos bx + nb\sin bx) + \frac{n(n-1)}{a^2+b^2n^2}b^2\int e^{ax}\cos^{n-2} bx\,dx$

（十四）含有对数函数的积分

132. $\int \ln x\,dx = x\ln x - x + C$

133. $\int \frac{dx}{x\ln x} = \ln(\ln x) + C$

134. $\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right] + C$

135. $\int \ln^n x\,dx = x\ln^n x - n\int \ln^{n-1} x\,dx$

136. $\int x^m \ln^n x\,dx = \frac{x^{m+1}}{m+1}\ln^n x - \frac{n}{m+1}\int x^m \ln^{n-1} x\,dx$

（十五）含有双曲函数的积分

137. $\int \sinh x\,dx = \cosh x + C$

138. $\int \cosh x\,dx = \sinh x + C$

139. $\int \tanh x\,dx = \ln\cosh x + C$

140. $\int \sinh^2 x\,dx = -\frac{x}{2} + \frac{1}{4}\sinh 2x + C$

141. $\int \cosh^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sinh 2x + C$

（十六）定　积　分

142. $\int_{-\pi}^{\pi} \cos nx\,dx = \int_{-\pi}^{\pi} \sin nx\,dx = 0$

143. $\int_{-\pi}^{\pi} \cos mx \sin nx\,dx = 0$

144. $\int_{-\pi}^{\pi} \cos mx \cos nx\,dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

145. $\int_{-\pi}^{\pi} \sin mx \sin nx\,dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

146. $\int_0^{\pi}\sin mx\sin nx\,dx=\int_0^{\pi}\cos mx\cos nx\,dx=\begin{cases}0, & m\neq n\\ \pi/2 & m=n\end{cases}$

147. $I_n=\int_0^{\frac{\pi}{2}}\sin^n x\,dx=\int_0^{\frac{\pi}{2}}\cos^n x\,dx$

$I_n=\dfrac{n-1}{n}I_{n-2}$

$$\begin{cases}I_n=\dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\cdots\cdot\dfrac{4}{5}\cdot\dfrac{2}{3} & (n\text{为大于1的正奇数}),I_1=1\\ I_n=\dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\cdots\cdot\dfrac{3}{4}\cdot\dfrac{1}{2}\cdot\dfrac{\pi}{2} & (n\text{为正偶数}),I_0=\dfrac{\pi}{2}\end{cases}$$

注1 该表中，凡$\int\frac{dx}{x}=\ln|x|+C$，皆省略了绝对值符号，而简写成$\int\frac{dx}{x}=\ln x+C$.

注2 该表中的记号 arsinh x 和 arcosh x 是指双曲函数 sinh x 和 cosh x 的反函数.